

SIMULTANEOUS HEAT AND MASS TRANSFER IN SPHERE IN PRESENCE OF PHASE AND CHEMICAL TRANSFORMATION UNDER GENERALISED BOUNDARY CONDITIONS

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Abstract—The present investigation includes a theoretical study of simultaneous heat and mass transfer in a sphere in presence of phase and chemical transformation. The transfer potentials are determined under the influence of most general type of boundary conditions. Approximate solutions of these transfer potentials have also been determined. The whole analysis has been presented in the dimensionless form with the help of similarity criteria.

NOMENCLATURE

a , coefficient of diffusivity;
 A_i, B_i , dimensionless known thermophysical coefficients;
 A_m , defined by equation (35a);
 A_{nj}, B_{nj} , defined by equation (22a) and (22b);
 c , specific content;
 D_{nj} , defined by equation (35b);
 F_0 , Fourier number;
 $f_i(x)$, given functions;
 fo_i , constant values of the given functions;
 k , reaction constant;
 n' , order of reaction;
 P_{nj} , defined by equation (18);
 Q_{nj} , defined by equation (19);
 Q_d , heat of reaction;
 r , space variable;
 R , radius of the sphere;
 s , Laplace parameter;
 t , time variable;
 T , temperature distribution;
 W , concentration of the matter;
 W_0 , dimensionless simplex;
 x , dimensionless space variable.

Greek symbols

μ_n, ν_m , characteristic root defined by (17) and (27);
 ψ_n , defined by equation (21);
 $\bar{\theta}_i$, Laplace transform of θ_i ;
 ε , phase criterion;
 ρ , specific heat of evaporation;
 δ , Soret coefficient.

Subscripts

q , heat;
 m , associated matter;
 d_1 , solid phase;
 d_2 , gaseous products of decomposition.

Superscript

o , characteristic entity.

Suffixes

i , 1, 2, 3, 4;
 j , 1, 2;
 m , 1, 2, ..., ∞ ;
 n , 1, 2, ..., ∞ .

INTRODUCTION

THE PRESENT paper deals with a theoretical study of simultaneous heat and mass transfer in a spherical capillary porous body in presence of phase and chemical transformation. Such type of phenomena occurs frequently during the kilning of ceramic ware and also kaoling kilning.

Thermal effects become apparent when a majority of mineral substances, etc. are treated thermally in different temperature ranges. Since the process of transfer becomes very complicated, much attention has not been paid to it by researchers in the field. However, the theoretical investigation carried out by Lebedev [1], Luikov and Mikhailov [2] on the one hand and the experimental work carried out by Ralko [3] on the other are noteworthy.

In the text of [2] the authors have studied simultaneous heat- and mass-transfer phenomena in an infinite plate in presence of phase and chemical transformation under the convective type of interaction law between the body surface and the gaseous medium. Previously the authors [4] considered the same system under the influence of the most generalized boundary conditions. The initial distributions of the transfer potentials and the chemical reactions have also been considered to be of the most general type.

The present work is in continuation of the author's previous work. Expressions for the transfer potentials are obtained in a sphere for the general type of chemical reaction and arbitrary initial distributions. The expressions are further simplified for a chemical reaction of the first order and for uniform initial distributions.

STATEMENT OF THE PROBLEM

The internal heat and mass exchange for unidimensional bodies in the form of a sphere may be described by the system of differential equations.

$$\frac{\partial T}{\partial t} = a_q \left(\frac{\partial^2 T}{\partial r^2} + \frac{2}{r} \frac{\partial T}{\partial r} \right) + \frac{\varepsilon \rho}{c_q} \frac{\partial W_m}{\partial t} - \frac{Q_d}{c_q} \frac{\partial W_d}{\partial t} \quad (1)$$

$$\frac{\partial W_m}{\partial t} = a_m \left(\frac{\partial^2 W_m}{\partial r^2} + \frac{2}{r} \frac{\partial W_m}{\partial r} \right) + a_m \delta_m \left(\frac{\partial^2 T}{\partial r^2} + \frac{2}{r} \frac{\partial T}{\partial r} \right) \quad (2)$$

and

$$\frac{\partial W_{d_1}}{\partial t} = a_{d_2} \left(\frac{\partial^2 W_{d_2}}{\partial r^2} + \frac{2}{r} \frac{\partial W_{d_2}}{\partial r} \right) + a_{d_2} \delta_{d_2} \left(\frac{\partial^2 T}{\partial r^2} + \frac{2}{r} \frac{\partial T}{\partial r} \right) - \frac{\partial W_{d_1}}{\partial t}, \quad 0 < r < R, \quad t > 0; \quad (3)$$

where $T = T(r, t)$ and $W = W(r, t)$ are the potential distributions of heat and matter respectively.

In the thermal decomposition of the body, the rate of chemical reaction depends upon the concentration of the reacting components and the products of decomposition. The rate of reaction to a first approximation is a function of the concentration of the reactants and thus

$$\frac{\partial W_{d_1}}{\partial t} = -k f_1(W_{d_1}) \quad (4)$$

where $f_1(W_{d_1})$ is some given function.

The system of differential equations (1)–(4) is transformed by using dimensionless variables

$$x = \frac{r}{R}, \quad F_0 = \frac{a_q t}{R^2}, \quad \theta_1 = \frac{T}{T^0}, \quad \theta_2 = \frac{W_m}{W_m^0}, \quad \theta_3 = \frac{W_{d_1}}{W_{d_1}^0} \quad \text{and} \quad \theta = \frac{W_{d_2}}{W_{d_2}^0}$$

and similarity criteria:

(i) The Luikov criteria of the field of bound matter and the products of decomposition in relation to temperature field

$$Lu_m = \frac{a_m}{a_q} \quad \text{and} \quad Lu_d = \frac{a_{d_2}}{a_q}$$

(ii) The Posnov criteria for bound matter and the gaseous products of decomposition

$$Pn_m = \frac{\delta_m T^0}{W_m^0} \quad \text{and} \quad Pn_d = \frac{\delta_{d_2} T^0}{W_{d_2}^0}$$

(iii) The Kossovich criteria for the bound matter and gaseous products of decomposition

$$Ko_m = \frac{\rho W_m^0}{c_q T^0} \quad \text{and} \quad Ko_d = \frac{Q_d W_{d_1}^0}{c_q T^0}$$

(iv) The Hess criterion

$$Ge = \frac{kR^2}{a_{d_2}} (W_{d_1}^0)^{n-1}$$

and the dimensionless simplex

$$W_0 = W_{d_1}^0 / W_{d_2}^0.$$

The system of differential equations (1)–(4) now becomes

$$\frac{\partial(x\theta_1)}{\partial F_0} = \frac{\partial^2(x\theta_1)}{\partial x^2} + \varepsilon Ko_m \frac{\partial(x\theta_2)}{\partial F_0} - Ko_d \frac{\partial(x\theta_3)}{\partial F_0}, \quad (5)$$

$$\frac{\partial(x\theta_2)}{\partial Fo} = Lu_m \frac{\partial^2(x\theta_2)}{\partial x^2} + Lu_m Pn_m \frac{\partial^2(x\theta_1)}{\partial x^2}, \quad (6)$$

$$\frac{\partial(x\theta_4)}{\partial Fo} = Lu_d \frac{\partial^2(x\theta_4)}{\partial x^2} + Lu_d Pn_d \frac{\partial^2(x\theta_1)}{\partial x^2} - \frac{1}{W_0} \frac{\partial(x\theta_3)}{\partial Fo} \quad (7)$$

and

$$\frac{\partial\theta_3}{\partial Fo} = -Lu_d Gef(\theta_3), \quad 0 < x < 1 \quad \text{and} \quad Fo > 0. \quad (8)$$

The boundary conditions for the system of differential equations (5)–(8) can be defined as

$$\theta_{1,x}(1, Fo) + A_1 \theta_1(1, Fo) + B_1 \theta_2(1, Fo) = \phi_1(Fo) \quad (9)$$

$$\theta_{2,x}(1, Fo) + A_2 \theta_{1,x}(1, Fo) + B_2 \theta_2(1, Fo) = \phi_2(Fo) \quad (10)$$

$$\theta_{4,x}(1, Fo) + A_3 \theta_{1,x}(1, Fo) + B_3 \theta_4(1, Fo) = \phi_3(Fo) \quad (11)$$

and

$$\theta_{i,x}(0, Fo) = 0 \quad (12)$$

where A_i and B_i ($i = 1, 2, 3$) are aggregates of known thermophysical coefficients in the dimensionless forms and $\phi_i(Fo)$, $i = 1, 2, 3$ are prescribed fluxes which are to be determined by the experiments. Subscript “ x ” stands for the partial differentiation with respect to x .

For a complete statement, we shall assume that the transfer potentials are arbitrary functions of space coordinate at the initial moment of time, i.e.

$$\theta_i(x, 0) = f_i(x); \quad (13)$$

where $f_i(x)$ are some known functions in the dimensionless form.

SOLUTION OF THE PROBLEM

The set of differential equations (5)–(7) through the boundary and initial conditions (9)–(13) are solved by the application of Laplace transform technique. The solutions under the Laplace transform can be put as

$$\begin{aligned} \bar{\theta}_1(x, s) = & \frac{1}{x(Q_1 P_2 - P_1 Q_2)s} \\ & \times \{ [s(Q_1 \sinh v_2(\sqrt{s})x - Q_2 \sinh v_1(\sqrt{s})x)\bar{\phi}_1(s) - (P_1 \sinh v_2(\sqrt{s})x - P_2 \sinh v_1(\sqrt{s})x)\bar{\phi}_2(s)] - (v_1^2 - v_2^2)^{-1} \\ & \times [(S_1 - S_2)(P_1 \sinh v_2(\sqrt{s})x - P_2 \sinh v_1(\sqrt{s})x) - (R_1 - R_2)(Q_1 \sinh v_2(\sqrt{s})x - Q_2 \sinh v_1(\sqrt{s})x)] \\ & + (1/\varepsilon K_{Om})[(F, x(1) + B_2 F(1))(P_1 \sinh v_2(\sqrt{s})x - P_2 \sinh v_1(\sqrt{s})x) + B_1 F(1)] \\ & \times (Q_2 \sinh v_1(\sqrt{s})x - Q_1 \sinh v_2(\sqrt{s})x) \} \\ & + \frac{1}{v_1 v_2 (v_1^2 - v_2^2) s^2 x} \int_0^x R(\xi) [v_2(\sqrt{s}) \sinh(x - \xi)v_1(\sqrt{s}) - v_1(\sqrt{s}) \sinh(x - \xi)v_2(\sqrt{s})] d\xi. \quad (14) \end{aligned}$$

and

$$\begin{aligned} \bar{\theta}_2(x, s) = & \frac{1}{\varepsilon K_{Om} x(Q_1 P_2 - P_1 Q_2)s} \{ [s(Q_1(1 - v_2^2) \sinh v_2(\sqrt{s})x - Q_2(1 - v_1^2) \sinh v_1(\sqrt{s})x)\bar{\phi}_1(s)] \\ & - (P_1(1 - v_2^2) \sinh v_2(\sqrt{s})x - P_2(1 - v_1^2) \sinh v_1(\sqrt{s})x)\bar{\phi}_2(s) + (v_1^2 - v_2^2)^{-1} \\ & \times [(S_1 - S_2)(P_1(1 - v_2^2) \sinh v_2(\sqrt{s})x - P_2(1 - v_1^2) \sinh v_1(\sqrt{s})x) \\ & - (R_1 - R_2)(Q_1(1 - v_2^2) \sinh v_2(\sqrt{s})x - Q_2(1 - v_1^2) \sinh v_1(\sqrt{s})x)] + (1/\varepsilon K_{Om}) \\ & \times [(1 - v_1^2)(P_1 \sinh v_2(\sqrt{s})x - P_2(1 - v_1^2) \sinh v_1(\sqrt{s})x)(F, x(1) + B_2 F(1)) \\ & + B_1 F(1)(Q_2(1 - v_1^2) \sinh v_1(\sqrt{s})x - Q_1(1 - v_2^2) \sinh v_2(\sqrt{s})x)] \} \\ & + \frac{1}{\varepsilon K_{Om} x(v_1^2 - v_2^2)x} \int_0^x \left\{ R(\xi) \frac{1 - v_1^2}{v_1(\sqrt{s})} \sinh(x - \xi)v_1(\sqrt{s}) \right. \\ & \left. - \frac{1 - v_2^2}{v_2(\sqrt{s})} \sinh(x - \xi)v_2(\sqrt{s}) \right\} d\xi + \frac{1}{\varepsilon K_{Om} x s} F(x), \quad (15) \end{aligned}$$

$$v_j^2 = \frac{1}{2} \left\{ \left(1 + \varepsilon K_{Om} Pn_m + \frac{1}{Lu_m} \right) + (-1)^j \sqrt{\left[\left(1 + \varepsilon K_{Om} Pn_m + \frac{1}{Lu_m} \right)^2 - \frac{4}{Lu_m} \right]} \right\}$$

and the other functions are defined as

$$R(x) = s/Lu_m [xf_1(x) + Lu_d Ko_d Ge xf(\theta_3)] - xf_1''(x) - 2f_1'(x) + Ko_m xf_2''(x) + 2Ko_m f_2'(x) - Lu_d Ge Ko_d x\bar{f}''(\theta_3) - 2Lu_d Ge Ko_d \bar{f}'(\theta_3),$$

$$F(x) = -xf_1(x) + \epsilon Ko_m xf_2(x) - Lu_d Ge Ko_d \bar{f}(\theta_3),$$

$$P_j = \left(-1 + A_1 + (1 - v_j^2) \frac{B_1}{\epsilon Ko_m} \right) \sinh v_j(\sqrt{s}) + v_j(\sqrt{s}) \cosh v_j(\sqrt{s})$$

$$Q_j = \left(A_2 + \frac{1 - v_j^2}{\epsilon Ko_m} \right) v_j(\sqrt{s}) \cosh v_j(\sqrt{s}) + \left((B_2 - 1) \frac{1 - v_j^2}{\epsilon Ko_m} - A_2 \right) \sinh v_j(\sqrt{s}),$$

$$R_j = \left(-1 + A_1 + (1 - v_j^2) \frac{B_1}{\epsilon Ko_m} \right) \frac{1}{v_j(\sqrt{s})} \int_0^1 R(\xi, s) \sinh(1 - \xi)v_j(\sqrt{s}) d\xi + \int_0^1 R(\xi, s) \cosh(1 - \xi)v_j(\sqrt{s}) d\xi$$

and

$$S_j = \left((B_2 - 1)(1 - v_j^2) \frac{1}{\epsilon Ko_m} - A_2 \right) \frac{1}{v_j(\sqrt{s})} \int_0^1 R(\xi, s) \sinh(1 - \xi)v_j(\sqrt{s}) d\xi + \left(\frac{1 - v_j^2}{\epsilon Ko_m} + A_2 \right) \int_0^1 R(\xi, s) \cosh(1 - \xi)v_j(\sqrt{s}) d\xi.$$

The expression for the transfer potential of the gaseous product is obtained by solving the differential equation (7) through the boundary condition (11) by the Laplace transform technique and thus

$$\begin{aligned} \bar{\theta}_4(s, x) &= \frac{\sinh(s/Lu_d)^{\frac{1}{2}} x}{x[(s/Lu_d)^{\frac{1}{2}} \cosh(s/Lu_d)^{\frac{1}{2}} + (B_3 - 1) \sinh(s/Lu_d)^{\frac{1}{2}}]} \\ &\times \left[\bar{\phi}_3(s) - \frac{B_3}{(sLu_d)^{\frac{1}{2}}} \int_0^1 [\xi f_4(\xi) + Lu_d Pn_d(\bar{\theta}_{1,xx}(\xi)) + 2Q_{1,x}(\xi)] \sinh(s/Lu_d)^{\frac{1}{2}} (\xi - 1) d\xi \right. \\ &- A_1 \bar{\theta}_{1,x} + (1/Lu_d) \int_0^1 [\xi f_4(\xi) + Lu Pn_d(\bar{\theta}_{1,xx}(\xi)) + 2\bar{\theta}_{1,x}(\xi) + (\xi/W_0) \bar{f}(\theta_3)] \\ &\times \left. \left(\frac{1}{(s/Lu_d)^{\frac{1}{2}}} \sinh(s/Lu_d)^{\frac{1}{2}} (\xi - 1) - \cosh(s/Lu_d)^{\frac{1}{2}} (\xi - 1) \right) d\xi \right] + \frac{1}{x(sLu_d)^{\frac{1}{2}}} \\ &\times \int_0^x [\xi f_4(\xi) + Lu_d Pn_d(\bar{\theta}_{1,xx}(\xi)) + 2\bar{\theta}_{1,x}(\xi)] + \frac{Lu_d Ge}{W_0} \bar{f}(\theta_3) \sinh(s/Lu_d)^{\frac{1}{2}} (\xi - x) d\xi; \quad (16) \end{aligned}$$

where the values of $\bar{\theta}_{1,x}$ and $\bar{\theta}_{1,xx}$ are determined by (14).

The expression for the functions $\theta_i(x, s)$ contains the terms $\bar{\phi}_i(s)$ and $\bar{f}(\theta_3)$, the true nature of which is not yet defined in the context. Therefore, to determine the inverted form of these expressions we shall apply.

- (i) The inversion theorem of the complex analysis, where the expressions contain all the well-defined terms.
- (ii) The convolution theorem for terms of $\bar{\phi}_i(s)$ and $\bar{f}(\theta_3)$.

To apply the inversion theorem, we shall suppose that the expression in the denominator has only simple roots and its degree is always greater than that of the expressions in the numerator. Now the zeros of the denominator are obtained from

$$\psi(s) = s(Q_1 P_2 - P_1 Q_2) = 0.$$

This gives

- (i) $s = s_0 = 0$ (a zero root)
- (ii) $s = s_n$, where s_n satisfies the equation:

$$Q_1 P_2 - P_1 Q_2 = 0$$

or more clearly

$$Q_{n1} P_{n2} - P_{n1} Q_{n2} = 0 \tag{17}$$

where the hyperbolic sines and cosines are changed into sine and cosine by substituting $s_n = -\mu_n^2$. The values of P_{nj} and Q_{nj} are given by

$$P_{nj} = \mu_n v_j \cos \mu_n v_j + \left(-1 + A_1 + (1 - v_j^2) \frac{B_1}{\varepsilon K O_m} \right) \sin \mu_n v_j \quad (18)$$

and

$$Q_{nj} = \left(A_2 + \frac{1 - v_j^2}{\varepsilon K O_m} \right) \mu_n v_j \cos \mu_n v_j + \left((B_2 - 1) \frac{1 - v_j^2}{\varepsilon K O_m} - A_2 \right) \sin \mu_n v_j. \quad (19)$$

For determination of the residue, we need the value of the derivative of the denominator at $s = -\mu_n^2$. This gives

$$\psi_0^1(s_n) = \frac{1}{2} \mu_n \psi_n; \quad (20)$$

where

$$\psi_n = v_1 Q_{n2} A_{n1} + v_2 P_{n1} A_{n2} - v_2 Q_{n1} A_{n2} - v_1 P_{n2} B_{n1}. \quad (21)$$

The quantities A_{nj} and B_{nj} are

$$A_{nj} = \left(A_1 - 2 + (1 - v_j^2) \frac{B_1}{\varepsilon K O_m} \right) \cos \mu_n v_j + \mu_n v_j \sin \mu_n v_j \quad (22a)$$

and

$$B_{nj} = \left(2A_2 - \frac{1}{\varepsilon K O_m} (B_2 - 1)(1 - v_j^2) \right) \cos \mu_n v_j - \left(A_2 + \frac{1 - v_j^2}{\varepsilon K O_m} \right) \mu_n v_j \sin \mu_n v_j. \quad (22b)$$

The inverted expression of the transfer potentials $\theta_i(x, Fo)$ can be written as

$$\begin{aligned} \theta_1(x, Fo) = & \frac{2}{x} \sum_{n=1}^{\infty} \frac{\mu_n}{\psi_n} \int_0^{Fo} \left[(Q_{n1} \sin \mu_n v_2 x - Q_{n2} \sin \mu_n v_1 x) \phi_1(u) - (P_{n1} \sin \mu_n v_2 x - P_{n2} \sin \mu_n v_1 x) \phi_2(u) \right] \\ & \times \exp(-\mu_n^2 \overline{Fo - u}) du + \frac{B_1}{A_1 B_2 \varepsilon K O_m} \left[f_1'(1) - \varepsilon K O_m f_2'(1) \right] + \frac{2}{(v_1^2 - v_2^2) x} \sum_{n=1}^{\infty} \frac{1}{\mu_n \psi_n} \\ & \times \left[(S_{n1}^* - S_{n2}^*) (P_{n1} \sin \mu_n v_2 x - P_{n2} \sin \mu_n v_1 x) - (R_{n1}^* - R_{n2}^*) (Q_{n1} \sin \mu_n v_2 x - Q_{n2} \sin \mu_n v_1 x) \right] \\ & \times \exp(-\mu_n^2 Fo) - \frac{1}{\varepsilon K O_m} \left[\frac{B_1}{A_1 B_2} + \frac{2}{x} \sum_{n=1}^{\infty} \frac{1}{\mu_n \psi_n} (P_{n1} \sin \mu_n v_2 x - P_{n2} \sin \mu_n v_1 x) \exp(-\mu_n^2 Fo) \right] \\ & \times [B_2 F_1(1) + F_{1,x}(1)] + \frac{B_1}{\varepsilon K O_m} \left[-\frac{1}{A_1} + \frac{2}{x} \sum_{n=1}^{\infty} \frac{1}{\mu_n \psi_n} (Q_{n2} \sin \mu_n v_1 x - Q_{n1} \sin \mu_n v_2 x) \exp(-\mu_n^2 Fo) \right] \\ & \times [F_1(1)] + \frac{Lu_d Ge K O_d}{v_1^2 - v_2^2} \int_0^{Fo} H_1(u) \\ & \times \left[\frac{B_1}{A_1 B_2} + \frac{2}{x} \sum_{n=1}^{\infty} \frac{1}{\mu_n \psi_n} (P_{n1} \sin \mu_n v_2 x - P_{n2} \sin \mu_n v_1 x) \exp(-\mu_n^2 \overline{Fo - u}) \right] du \\ & + \frac{Lu_d Ge K O_d}{v_1^2 - v_2^2} \int_0^{Fo} H_2(u) \left[-\frac{1}{A_1} + \frac{2}{x} \sum_{n=1}^{\infty} \frac{1}{\mu_n \psi_n} (Q_{n2} \sin \mu_n v_1 x - Q_{n1} \sin \mu_n v_2 x) \exp(-\mu_n^2 \overline{Fo - u}) \right] du \\ & + \frac{Lu_d Ge K O_d}{\varepsilon K O_m} \int_0^{Fo} H_3(u) \left[\frac{B_1}{A_1 B_2} + \frac{2}{x} \sum_{n=1}^{\infty} \frac{1}{\mu_n \psi_n} (P_{n1} \sin \mu_n v_2 x - P_{n2} \sin \mu_n v_1 x) \exp(-\mu_n^2 \overline{Fo - u}) \right] du \quad (23) \end{aligned}$$

$$\begin{aligned} \theta_2(x, Fo) = & \frac{2}{\varepsilon K O_m x} \sum_{n=1}^{\infty} \frac{\mu_n}{\psi_n} \int_0^{Fo} \left\{ [Q_{n1}(1 - v_2^2) \sin \mu_n v_2 x - Q_{n2}(1 - v_1^2) \sin \mu_n v_1 x] \phi_1(u) \right. \\ & \left. - [P_{n1}(1 - v_2^2) \sin \mu_n v_2 x - P_{n2}(1 - v_1^2) \sin \mu_n v_1 x] \phi_2(u) \right\} \exp(-\mu_n^2 \overline{Fo - u}) du \\ & + \frac{1}{\varepsilon K O_m} \left[f_1(1) - f_1(0) + \varepsilon K O_m f_2(1) - \varepsilon K O_m f_2(0) - \frac{1}{B_2} f_1'(1) + \frac{\varepsilon K O_m}{B_2} f_2'(1) \right] + \frac{2}{\varepsilon K O_m} \frac{1}{(v_1^2 - v_2^2) x} \\ & \times \sum_{n=1}^{\infty} \frac{1}{\mu_n \psi_n} \left\{ (S_{n1}^* - S_{n2}^*) [P_{n1}(1 - v_2^2) \sin \mu_n v_2 x - P_{n2}(1 - v_1^2) \sin \mu_n v_1 x] - (R_{n1}^* - R_{n2}^*) \right. \\ & \times [Q_{n1}(1 - v_2^2) \sin \mu_n v_2 x - Q_{n2}(1 - v_1^2) \sin \mu_n v_1 x] \left. \right\} \exp(-\mu_n^2 Fo) \\ & + \frac{2}{\varepsilon^2 K O_m^2 x} \sum_{n=1}^{\infty} \frac{1}{\mu_n \psi_n} [(1 - v_2^2) P_{n1} \sin \mu_n v_2 x - (1 - v_1^2) P_{n2} \sin \mu_n v_1 x] \exp(-\mu_n^2 Fo) [B_2 F_1(1) + F_{1,x}(1)] \end{aligned}$$

$$\begin{aligned}
 & -\frac{2B_1}{\varepsilon^2 K_o m^2 x} \sum_{n=1}^{\infty} \frac{F_1(1)}{\mu_n \psi_n} [(1-v_1^2)Q_{n2} \sin \mu_n v_1 x - (1-v_2^2)Q_{n1} \sin \mu_n v_2 x] \exp(-\mu_n^2 Fo) \\
 & + \frac{Lu_d Ge K_o d}{(v_1^2 - v_2^2) \varepsilon K_o m x} \int_0^{Fo} H_1(Fo - u) \\
 & \times \left[\frac{\varepsilon K_o m x}{B_2} - 2 \sum_{n=1}^{\infty} \frac{1}{\mu_n \psi_n} [P_{n1}(1-v_2^2) \sin \mu_n v_2 x - P_{n2}(1-v_1^2) \sin \mu_n v_1 x] \exp(-\mu_n^2 u) \right] du \\
 & + \frac{2Lu_d Ge K_o d}{(v_1^2 - v_2^2) \varepsilon K_o m x} \int_0^{Fo} H_2(Fo - u) \sum_{n=1}^{\infty} \frac{1}{\mu_n \psi_n} [Q_{n1}(1-v_2^2) \sin \mu_n v_2 x - Q_{n2}(1-v_1^2) \sin \mu_n v_1 x] \exp(-\mu_n^2 u) du \\
 & - \frac{Lu_d Ge K_o d}{\varepsilon K_o m x} \int_0^{Fo} \int_0^x H_5(\zeta, u)(x - \zeta) du d\zeta + \frac{Lu_d Ge K_o d}{\varepsilon^2 K_o m^2 x} \int_0^{Fo} H_3(Fo - u) \\
 & \times \left[\frac{\varepsilon K_o m x}{B_2} - 2 \sum_{n=1}^{\infty} \frac{1}{\mu_n \psi_n} [P_{n1}(1-v_2^2) \sin \mu_n v_2 x - P_{n2}(1-v_1^2) \sin \mu_n v_1 x] \exp(-\mu_n^2 u) \right] du \\
 & + 2 \frac{Lu_d Ge K_o d}{\varepsilon^2 K_o m^2 x} B_1 \int_0^{Fo} H_4(Fo - u) \sum_{n=1}^{\infty} \frac{1}{\mu_n \psi_n} [Q_{n2}(1-v_1^2) \sin \mu_n v_1 x - Q_{n1}(1-v_2^2) \sin \mu_n v_2 x] \exp(-\mu_n^2 u) du \\
 & + \frac{1}{\varepsilon K_o m x} H_6(x, Fo) - \frac{1}{\varepsilon K_o m} \left[f_1(x) - \varepsilon K_o m f_2(x) - \int_0^x f_1(\xi) d\xi + \varepsilon K_o m \int_0^x f_2(\xi) d\xi \right]. \tag{24}
 \end{aligned}$$

Equation (8) can directly be integrated as

$$\theta_3(x, Fo) - f_3(x) = -Lu_d Ge \int_0^{Fo} f(\theta_3) du. \tag{25}$$

To obtain the inverted expression of $\bar{\theta}_4(x, s)$, we have to obtain the roots of the equation

$$(s/Lu_d)^{\frac{1}{2}} \cosh(s/Lu_d)^{\frac{1}{2}} + (B_3 - 1) \sinh(s/Lu_d)^{\frac{1}{2}} = 0, \tag{26}$$

which has an infinite set of simple zeros at $s = s_m$. For a more precise value of s_m , changing the hyperbolic sine and cosine into ordinary sine and cosine, we obtain

$$s_m = -v_m^2;$$

where v_m are the roots of the characteristic equation

$$-\frac{v}{(Lu_d)^{\frac{1}{2}}} \cos \frac{v}{(Lu_d)^{\frac{1}{2}}} + (B_3 - 1) \sin \frac{v}{(Lu_d)^{\frac{1}{2}}} = 0 \tag{27}$$

and thus the inverted expression for the transfer potential $\bar{\theta}_4(x, s)$ becomes

$$\begin{aligned}
 \theta_4(x, Fo) = & -2(Lu_d)^{\frac{1}{2}} \int_0^{Fo} \sum_{n=1}^{\infty} \frac{v_m \sin[v_m/(Lu_d)^{\frac{1}{2}} x]}{x \{ B_3 \cos[v_m/(Lu_d)^{\frac{1}{2}}] [v_m/(Lu_d)^{\frac{1}{2}}] \sin[v_m/(Lu_d)^{\frac{1}{2}}] \}} \\
 & \times \exp(-v_m^2 Fo - u) \phi_3(u) du - \frac{2}{x} \int_0^{Fo} \sum_{m=1}^{\infty} \xi B(\xi) f_4(\xi) \exp(-v_m^2 Fo) d\xi \\
 & - \frac{2Lu_d}{x} P_{n_d} \sum_{m=1}^{\infty} \int_0^{Fo} \int_0^1 B(\xi) \sin[v_m/(Lu_d)^{\frac{1}{2}} x] [\bar{\theta}_{1,xx}(\xi) + 2\bar{\theta}_{1,x}(\xi)] \\
 & \times \exp(-v_m^2 Fo - u) d\xi du - 2 \frac{\xi}{x W_o} \sum_{m=1}^{\infty} \int_0^{Fo} \int_0^1 B(\xi) f(\theta_3) du d\xi + \frac{1}{x} \int_0^x G(x, \xi, Fo) d\xi; \tag{28}
 \end{aligned}$$

where

$$\begin{aligned}
 R_{nj}^* = & \left(-1 + A_1 + (1-v_j^2) \frac{B_1}{\varepsilon K_o m} \right) \frac{1}{\mu_n v_j} \int_0^1 R^*(\xi) \sin[(1-\xi)\mu_n v_j] d\xi + \int_0^1 R^*(\xi) \cos[(1-\xi)\mu_n v_j] d\xi, \\
 S_{nj}^* = & \left((B_2 - 1)(1-v_j^2) \frac{1}{\varepsilon K_o m} - A_2 \right) \frac{1}{\mu_n v_j} \int_0^1 R^*(\xi) \sin[(1-\xi)\mu_n v_j] d\xi + \left(\frac{1-v_j^2}{\varepsilon K_o m} + A_2 \right) \int_0^1 R^*(\xi) \cos[(1-\xi)\mu_n v_j] d\xi, \\
 R^*(\xi) = & -\frac{\mu_n^2}{Lu_m} \xi f_1(\xi) - \xi f_1''(\xi) - 2f_1''(\xi) - 2f_1'(\xi) + \varepsilon K_o m \xi f_2''(\xi) + 2\varepsilon K_o m f_2''(\xi)
 \end{aligned}$$

$$B(\xi) = \{(1 - B_3) \sin[v_m/(Lu_d)^{\frac{1}{2}}](\xi - 1)\} - [v_m/(Lu_d)^{\frac{1}{2}}] \cos\{[v_m/(Lu_d)^{\frac{1}{2}}](\xi - 1)\} \\ \times \{B_3 \cos[v_m/(Lu_d)^{\frac{1}{2}}] - [v_m/(Lu_d)^{\frac{1}{2}}] \sin[v_m/(Lu_d)^{\frac{1}{2}}]\}$$

$$H_1(Fo) = L^{-1}[\bar{g}_1(s) - \bar{g}_2(s)], \quad H_2(Fo) = L^{-1}[\bar{h}_1(s) - \bar{h}_2(s)],$$

$$H_3(Fo) = L^{-1}[\bar{f}_{1,x}(\theta_3) - (1 + B_3)\bar{f}(\theta_3)], \quad H_4(Fo) = L^{-1}\{-[\bar{f}(\theta_3)]_{x=1}\},$$

$$H_5(Fo) = L^{-1}[(s/Lu_m)x\bar{f}(\theta_3) - x\bar{f}''(\theta_3) - 2\bar{f}'(\theta_3)],$$

$$H_6(Fo) = -Lu_d GeKo_d L^{-1}\left[\frac{1}{s}\bar{f}(\theta_3)\right];$$

$$G(Fo) = L^{-1}\left\{(Lu_d/s)^{\frac{1}{2}}\left[Pn_d[\bar{\theta}_{1,xx}(\xi) + \xi\bar{\theta}_{1,x}(\xi)] + \xi\frac{Ge}{Wo}\bar{f}(\theta_3)\right]\right\},$$

$$\bar{g}_j(s) = \left((B_2 - 1)(1 - v_j^2)\frac{1}{\varepsilon Ko_m} - A_2\right)\frac{1}{v_j(s)^{\frac{1}{2}}}\int_0^1\left(\xi\bar{f}(\theta_3)\frac{s}{Lu_m} - \xi\bar{f}''(\theta_3) - 2\bar{f}'(\theta_3)\right)\sinh(1 - \xi)v_j(s)^{\frac{1}{2}}d\xi \\ + \left(A_2 + \frac{1 - v_j^2}{\varepsilon Ko_m}\right)\int_0^1\left(\xi\bar{f}(\theta_3)\frac{s}{Lu_m} - \xi\bar{f}''(\theta_3) - 2\bar{f}'(\theta_3)\right)\cosh(1 - \xi)v_j(s)^{\frac{1}{2}}d\xi;$$

$$\bar{h}_j(s) = \left(1 + A_1 + (1 - v_j^2)\frac{B_1}{\varepsilon Ko_m}\right)\frac{1}{v_j(s)^{\frac{1}{2}}}\int_0^1\left(\xi\bar{f}(\theta_3)\frac{s}{Lu_m} - \xi\bar{f}''(\theta_3) - 2\bar{f}'(\theta_3)\right)\sinh(1 - \xi)v_j(s)^{\frac{1}{2}}d\xi \\ + \int_0^1\left(\frac{s}{Lu_m}\xi\bar{f}(\theta_3) - \bar{f}''(\theta_3) - 2\bar{f}'(\theta_3)\right)\cosh(1 - \xi)v_j(s)^{\frac{1}{2}}d\xi$$

and

$$F_1(x, s) = -xf_1(x) + \varepsilon Ko_m f_2(x).$$

ANALYSIS OF THE RESULT

For a n^* th order chemical reaction equation (8) becomes

$$\frac{\partial\theta_3(x, Fo)}{\partial Fo} = -Lu_d Ge[\theta_3(x, Fo)]^{n^*}, \quad n^* > 0. \quad (29)$$

The solution (25) is thus modified as under:

$$\theta_3(x, Fo) = f_3(x)[1 - (n^* - 1)Lu_d GeFo f_3^{(n^*-1)}(x)]^{-(n^*-1)^{-1}}, \quad n^* \neq 1 \quad (30)$$

and

$$\theta_3(x, Fo) = f_3(x) + \exp(-Lu_d GeFo), \quad n^* = 1. \quad (31)$$

Generally it is found that the chemical reactions of order greater than two are rare, and, in various power plants, the chemical reaction of order one takes place frequently. Therefore it is of common interest to reduce the complicated expressions (23), (24) and (28) for the first order chemical reaction. Further we shall suppose that the transfer potentials are initially uniform:

$$\theta_i(x, 0) = f_{oi}(\text{constant}).$$

Therefore, the expressions (26), (27) and (31) reduce to

$$\theta_1(x, Fo) = \frac{2}{x}\sum_{n=1}^{\infty}\frac{\mu_n}{\psi_n}\int_0^{Fo}\left[(P_{n2}\sin\mu_n v_1 x - P_{n1}\sin\mu_n v_2 x)\phi_2(Fo - u) - (Q_{n2}\sin\mu_n v_1 x - Q_{n1}\sin\mu_n v_2 x)\phi_1(Fo - u)\right] \\ \times \exp(-\mu_n^2 u)du - \frac{2}{x}\sum_{n=1}^{\infty}\frac{1}{\mu_n\psi_n} \\ \times \left\{[(P_{n2}B_2 - B_1Q_{n2})f_{o2} - Q_{n2}A_1f_{o1}]\sin\mu_n v_1 x + [A_1Q_{n1}f_{o1} - (B_1Q_{n1} - B_2P_{n1})f_{o2}]\sin\mu_n v_2 x\right\} \\ \times \exp(-\mu_n^2 Fo) - Ko_d\left\{1 + [A_1/x(\bar{Q}_2\bar{P}_1 - \bar{P}_2\bar{Q}_1)]\right. \\ \left.\times [\bar{Q}_1\sin(Lu_d Ge)^{\frac{1}{2}}v_2 x - \bar{Q}_2\sin(Lu_d Ge)^{\frac{1}{2}}v_1 x]\exp(-Lu_d GeFo)\right\} \\ + 2A_1Lu_d GeKo_d\sum_{n=1}^{\infty}(1/\mu_n\psi_n)(Q_{n2}\sin\mu_n v_1 x - Q_{n1}\sin\mu_n v_2 x)/[x(Lu_d Ge - \mu_n^2)]\exp(-\mu_n^2 Fo); \quad (32)$$

$$\begin{aligned} \theta_2(x, Fo) = & \frac{2}{\varepsilon K O_m x} \sum_{n=1}^{\infty} \frac{\mu_n}{\psi_n} \int_0^{Fo} \{ [P_{n2}(1-v_1^2) \sin \mu_n v_1 x - (1-v_2^2) P_{n1} \sin \mu_n v_2 x] \phi_2(Fo-u) \\ & - [Q_{n2}(1-v_1^2) \sin \mu_n v_1 x - Q_{n1}(1-v_2^2) \sin \mu_n v_2 x] \phi_1(Fo-u) \} \\ & \times \exp(-\mu_n^2 u) du - \frac{2}{\varepsilon K O_m x} \sum_{n=1}^{\infty} \frac{1}{\mu_n \psi_n} [(P_{n2} B_2 - Q_{n2} B_1) f_{o2} - (1-v_2^2) \sin \mu_n v_2 x] \\ & \times \exp(-\mu_n^2 Fo) - A_1 [K O_d / x (\tilde{Q}_2 \tilde{P}_1 - \tilde{Q}_1 \tilde{P}_2)] [\tilde{Q}_1 (1-v_2^2) \sin(Lu_d Ge)^{\frac{1}{2}} v_2 x] - \tilde{Q}_2 (1-v_2^2) \\ & \times \sin[(Lu_d Ge)^{\frac{1}{2}} v_1 x] \exp(-Lu_d Ge Fo) + 2A_1 Lu_d Ge K O_d \sum_{n=1}^{\infty} [\mu_n \psi_n (Lu_d Ge - \mu_n^2) x]^{-1} \\ & \times [Q_{n2}(1-v_1^2) \sin \mu_n v_1 x - Q_{n1}(1-v_2^2) \sin \mu_n v_2 x] \exp(-\mu_n^2 Fo), \quad (33) \end{aligned}$$

$$\theta_3(x, Fo) - f_{o3} = \exp(-Lu_d Ge Fo) \quad (34)$$

$$\begin{aligned} \theta_4(x, Fo) = & -1/x \sum_{m=1}^{\infty} \int_0^{Fo} A_m \sin[v_m / (Lu_d)^{\frac{1}{2}} x] \exp(-v_m^2 \overline{Fo-u}) \phi_3(u) du \\ & - \frac{B_3 f_{o4}}{x} \sum_{m=1}^{\infty} (A_m / v_m^2) \sin[x v_m / (Lu_d)^{\frac{1}{2}}] \exp(-v_m^2 Fo) \\ & + (B_3 / W_0) \{ \sin(Ge)^{\frac{1}{2}} x / x [(Ge)^{\frac{1}{2}} \cos(Ge)^{\frac{1}{2}} + (B_3 - 1) \sin(Ge)^{\frac{1}{2}}] \} \\ & \times \exp(-Lu_d Ge Fo) - (1/W_0) \exp(-Lu_d Ge Fo) - (Lu_d Ge B_3 / W_0 x) \\ & \times \sum_{m=1}^{\infty} A_m / [v_m^2 (Lu_d Ge - v_m^2)] \sin[v_m x / (Lu_d)^{\frac{1}{2}}] \exp(-v_m^2 Fo) \\ & + \frac{2}{x} \sum_{n=1}^{\infty} (\mu_n / \psi_n) \int_0^{Fo} [(P_{n1} D_{n2} - P_{n2} D_{n1}) \phi_2(u) - (Q_{n1} D_{n2} - Q_{n2} D_{n1}) \phi_1(u)] \\ & \times (P_{m1} Q_{m2} - Q_{m1} P_{m2})^{-1} \sin[v_m x / (Lu_d)^{\frac{1}{2}}] \exp(-v_m^2 \overline{Fo-u}) du \\ & - \frac{2}{x} \sum_{n=1}^{\infty} (1/\mu_n \psi_n) [(B_2 P_{n2} - Q_{n2} B_1) D_{n1} f_{o2} + (Q_{n1} B_1 - P_{n1} B_2) D_{n2} f_{o2} - A_1 f_{o1} (Q_{n2} D_{n1} - Q_{n1} D_{n2})] \\ & \times \{ [\mu_n / (Lu_d)^{\frac{1}{2}}] \cos[\mu_n / (Lu_d)^{\frac{1}{2}}] + (B_3 - 1) \sin[\mu_n / (Lu_d)^{\frac{1}{2}}] \}^{-1} \exp(-\mu_n^2 Fo) \\ & + \frac{1}{x} \sum_{m=1}^{\infty} (A_m / v_m^2) [(B_2 P_{m2} - B_1 Q_{m2}) f_{o2} D_{m1} + (Q_{m1} B_1 - P_{m1} B_2) f_{o2} D_{m2} - (Q_{m2} D_{m1} - Q_{m1} D_{m2}) A_1 f_{o1}] \\ & \times (P_{m1} Q_{m2} - Q_{m1} P_{m2})^{-1} \exp(-v_m^2 Fo) + (2/x) Lu_d Ge K O_d A_1 \sum_{n=1}^{\infty} \frac{1}{\mu_n \psi_n} \\ & \times (Q_{n1} D_{n2} - Q_{n2} D_{n1}) \sin[\mu_n x / (Lu_d)^{\frac{1}{2}}] \exp(-\mu_n^2 Fo) \\ & \times \{ [\mu_n / (Lu_d)^{\frac{1}{2}}] \cos[\mu_n / (Lu_d)^{\frac{1}{2}}] + (B_3 - 1) \sin[\mu_n / (Lu_d)^{\frac{1}{2}}] \}^{-1} - (1/x) Lu_d Ge K O_d A_1 \\ & \times \sum_{m=1}^{\infty} (A_m / v_m^2) (Q_{m2} D_{m1} - Q_{m1} D_{m2}) (P_{m1} Q_{m2} - Q_{m1} P_{m2})^{-1} \sin[v_m x / (Lu_d)^{\frac{1}{2}}] \exp(-v_m^2 Fo). \\ & - (1/x) A_1 K O_d (\tilde{Q}_2 \tilde{D}_1 - \tilde{Q}_1 \tilde{D}_2) (\tilde{P}_1 \tilde{Q}_2 - \tilde{P}_2 \tilde{Q}_1)^{-1} \sin[(Ge)^{\frac{1}{2}} x] [(Ge)^{\frac{1}{2}} \cos(Ge)^{\frac{1}{2}} + (B_3 - 1) \sin(Ge)^{\frac{1}{2}}]^{-1} \\ & \times \exp(-Lu_d Ge Fo) + 2Pn_d \sum_{n=1}^{\infty} (\mu_n / \psi_n) \\ & \times \int_0^{Fo} \{ [(P_{n1} v_2^2 / (v_2^2 - 1/Lu_d)) \sin \mu_n v_2 x - (P_{n2} v_1^2 / (v_1^2 - 1/Lu_d)) \sin \mu_n v_1 x] \phi_2(u) \\ & - [(Q_{n1} v_2^2 / (v_2^2 - 1/Lu_d)) \sin \mu_n v_2 x - (Q_{n2} v_1^2 / (v_1^2 - 1/Lu_d)) \sin \mu_n v_1 x] \phi_1(u) \} \\ & \times \exp(-\mu_n^2 \overline{Fo-u}) du + 2Pn_d \sum_{n=1}^{\infty} (\mu_n / \psi_n) \\ & \times \{ [(B_2 P_{n2} - B_1 Q_{n2}) f_{o2} - A_1 f_{o1} Q_{n2}] [v_1^2 / (v_1^2 - 1/Lu_d)] \sin \mu_n v_1 x \\ & + [(B_1 Q_{n1} - B_2 P_{n1}) f_{o2} + A_1 f_{o1} Q_{n1}] [v_2^2 / (v_2^2 - 1/Lu_d)] \sin \mu_n v_2 x \} \exp(-\mu_n^2 Fo) \\ & + 2A_1 Lu_d Ge K O_d Pn_d \sum_{n=1}^{\infty} (1/\mu_n \psi_n) \{ [Q_{n2} v_1^2 / (v_1^2 - 1/Lu_d)] \sin \mu_n v_1 x - [Q_{n1} v_2^2 / (v_2^2 - 1/Lu_d)] \sin \mu_n v_2 x \} \\ & \times (Lu_d Ge - \mu_n^2) \exp(-\mu_n^2 Fo) + A_1 Pn_d K O_d \end{aligned}$$

$$\times \sum_{n=1}^{\infty} \{[\tilde{Q}_2 v_1^2/(v_1^2 - 1/Lu_d)] \sin(Lu_d Ge)^{\frac{1}{2}} v_1 x - [\tilde{Q}_1 v_2^2/(v_2^2 - 1/Lu_d)] \sin(Lu_d Ge)^{\frac{1}{2}} v_2 x\} (\tilde{P}_1 \tilde{Q}_2 - \tilde{P}_2 \tilde{Q}_1)^{-1} \\ \times \exp(-Lu_d Ge Fo) + [B_3 Lu_d (v_1^2 - 1/Lu_d)(v_2^2 - 1/Lu_d)]^{-1} Pn_d \epsilon K o_m f o_2, \quad (35)$$

where

$$A_m \equiv 2v_m (Lu_d)^{\frac{1}{2}} \{B_3 \cos[v_m/(Lu_d)^{\frac{1}{2}}] - [v_m/(Lu_d)^{\frac{1}{2}}] \sin[v_m/(Lu_d)^{\frac{1}{2}}]\}^{-1} \quad (35a)$$

and

$$D_{nj} = Pn_d (B_3 \sin \mu_n v_j + \mu_n v_j \cos \mu_n v_j) v_j^2 / [v_1^2 - 1(Lu_d)^{\frac{1}{2}}] + A_3 (\mu_n v_j \cos \mu_n v_j - \sin \mu_n v_j). \quad (35b)$$

P_{mj} , Q_{mj} , D_{mj} and \tilde{P}_j , \tilde{Q}_j , \tilde{D}_j are written in the same form as P_{nj} , Q_{nj} , D_{nj} ; the characteristic root being replaced by v_m and $(Lu_d Ge)^{\frac{1}{2}}$ respectively.

Since the quantity of gaseous products of decomposition in the system is small in comparison to the quantity of the associated matter and the transfer of matter through the gaseous products is also inconsiderable, the term Pn_d and the corresponding thermophysical coefficient A_3 have no importance. In this case the expression for the potentials of heat (32) and for the associated matter (33) remain unchanged and the expression for gaseous products (35) takes the form:

$$\theta_4(x, Fo) = -\frac{1}{x} \sum_{m=1}^{\infty} \int_0^{Fo} A_m \sin[v_m x/(Lu_d)^{\frac{1}{2}}] \exp(-v_m^2 Fo - u) \phi_3(u) du - \frac{B_3 f o_4}{x} \sum_{m=1}^{\infty} (A_m/v_m^2) \sin[v_m x/(Lu_d)^{\frac{1}{2}}] \\ \times \exp(-v_m^2 Fo) + \frac{Lu_d Ge}{W_0 x} B_3 \sum_{m=1}^{\infty} (A_m/v_m^2) (Lu_d Ge - v_m^2)^{-1} \sin[v_m x/(Lu_d)^{\frac{1}{2}}] \exp(-v_m^2 Fo) \\ + \frac{1}{W_0} B_3 \left[\frac{\sin(Ge)^{\frac{1}{2}} x}{x[(Ge)^{\frac{1}{2}} \cos(Ge)^{\frac{1}{2}} + (B_3 - 1) \sin(Ge)^{\frac{1}{2}}]} - 1 \right] \exp(-Lu_d Ge Fo). \quad (36)$$

The expressions for transfer potentials $\theta_i(x, Fo)$ contain a convergent series. Further, as the generalized time Fo increases, the terms of the series containing the exponential function of the criterion Fo diminish rapidly. For a certain value of $Fo > Fo_1$, the nature of the transfer potentials is mostly determined by retaining only first two terms of the series.

For the constant prescribed fluxes, i.e. $\phi_1(Fo) = Ki_q$, $\phi_2(Fo) = Ki_m$ and $\phi_3(Fo) = Ki_d$, where Ki are the Kirpichev criteria for the exchange of heat, matter and gaseous products respectively, the expressions (32), (33) and (36) reduce to:

$$\theta_1(x, Fo) = \frac{B_2 Ki_q - B_1 Ki_m}{A_1 B_2} - \sum_{n=1}^{\infty} \sum_{j=1}^2 \frac{E_{nj}}{x} \sin \mu_n v_j x \exp(-\mu_n^2 Fo) \\ - K o_d \left[1 + A_1 \frac{\tilde{Q}_1 \sin(Lu_d Ge)^{\frac{1}{2}} v_2 x - \tilde{Q}_2 \sin(Lu_d Ge)^{\frac{1}{2}} v_1 x}{x(\tilde{Q}_2 \tilde{P}_1 - \tilde{P}_2 \tilde{Q}_1)} \right] \exp(-Lu_d Ge Fo) \quad (37)$$

$$\theta_2(x, Fo) = \frac{Ki_m}{B_2} - \sum_{n=1}^{\infty} \sum_{j=1}^2 \frac{E_{nj}}{x} (1 - v_j^2) \sin \mu_n v_j x \exp(-\mu_n^2 Fo) \\ - A_1 K o_d \frac{\tilde{Q}_1 (1 - v_2^2) \sin(Lu_d Ge)^{\frac{1}{2}} v_2 x - \tilde{Q}_2 (1 - v_1^2) \sin(Lu_d Ge)^{\frac{1}{2}} v_1 x}{x(\tilde{Q}_2 \tilde{P}_1 - \tilde{P}_2 \tilde{Q}_1)} \exp(-Lu_d Ge Fo) \quad (38)$$

$$\theta_3(x, Fo) - f o_3 = \exp(-Lu_d Ge Fo) \quad (39)$$

and

$$\theta_4(x, Fo) = \frac{Ki_d}{B_3} + \frac{1}{x} \sum_{m=1}^{\infty} \frac{A_m}{v_m^2} \left[Ki_d - B_3 f o_4 - \frac{Lu_d Ge B_3}{W_0 (Lu_d Ge - v_m^2)} \right] \sin \frac{v_m}{(Lu_d)^{\frac{1}{2}}} x \exp(-v_m^2 Fo) \\ - \frac{1}{W_0} \left[1 - B_3 \frac{\sin(Ge)^{\frac{1}{2}} x}{x[(Ge)^{\frac{1}{2}} \cos(Ge)^{\frac{1}{2}} + (B_3 - 1) \sin(Ge)^{\frac{1}{2}}]} \right] \exp(-Lu_d Ge Fo); \quad (40)$$

where

$$E_{n1} = \frac{1}{\mu_n \psi_n} \left[P_{n2} (Ki_m - B_2 Fo_2) + Q_{n2} (A_1 f o_1 + B_1 f o_2 - Ki_q) - A_1 \frac{Lu_d Ge K o_d}{Lu_d Ge - \mu_n^2} Q_{n2} \right]$$

and

$$E_{n_2} = -\frac{2}{\mu_n \psi_n} \left[P_{n1}(Ki_m - B_2 f_{o_2}) + Q_{n1}(A_1 f_{o_1} + B_1 f_{o_2} - Ki_q) - A_1 \frac{Lu_d Ge Ko_d}{Lu_d Ge - \mu_n^2} Q_{n1} \right].$$

The mean values of the transfer potentials in case of the sphere are obtained from the relation

$$\langle \theta_1(Fo) \rangle = 3 \int_0^1 \theta_1(x, Fo) dx. \quad (41)$$

The characteristic roots v_m only affect the transfer potentials of the gaseous products $\theta_4(x, Fo)$ and the characteristic roots μ_n affect the transfer potentials of heat and matter. Since the gaseous products are formed only due to chemical reaction, the quantities Ki_{d_2} , v_m , B_3 and W_0 can be associated with the process of chemical reaction. The quantities Lu_d and Ge come together in the expressions of the transfer potentials of heat and matter and they govern the rate of chemical reaction; so they are responsible for the transfer of heat and mass due to thermal destruction. The quantity Ko_d is the Kossovich criterion for the gaseous products and it defines the nature of the chemical reaction. For $Ko_d > 0$, it signifies that the reaction is endothermic and proceeds with the absorption of heat and for $Ko_d < 0$ it signifies that the reaction is of exothermic type and proceeds with the evolution of heat.

Now we shall consider the approximate solutions for these transfer potentials applicable for small value of the generalized time, Fo . At small values of Fo , we have

$$\sinh v_j s^{\frac{1}{2}} \approx \cosh v_j s^{\frac{1}{2}} \approx \frac{1}{2} \exp(v_j s^{\frac{1}{2}}).$$

Under these approximations and restricting up to the terms of order $s^{3/2}$ only, the expressions (14)–(16) for uniform initial distribution of transfer potentials give

$$\begin{aligned} \theta_1(x, Fo) - f_{o_1} &= \frac{2}{Mx} \{ (N_{12} f_{o_2} - A_1 N_{22} f_{o_1})(Fo)^{\frac{1}{2}} i \operatorname{erfc} [v_1 \sqrt{1-x}/2(Fo)^{\frac{1}{2}}] - (N_{11} F_{o_2} - N_{21} A_1 f_{o_1})(Fo)^{\frac{1}{2}} \\ &\quad \times i \operatorname{erfc} [v_2 \sqrt{1-x}/2(Fo)^{\frac{1}{2}}] + (N_{31} Ki_m - N_{21} Ki_q)(Fo)^{\frac{1}{2}} \\ &\quad \times i \operatorname{erfc} [v_2 \sqrt{1-x}/2(Fo)^{\frac{1}{2}}] - (N_{32} Ki_m - N_{22} Ki_q)(Fo)^{\frac{1}{2}} \\ &\quad \times i \operatorname{erfc} [v_1 \sqrt{1-x}/2(Fo)^{\frac{1}{2}}] \} + Ko_d [1 - \exp(-Lu_d Ge Fo)] \quad (42) \end{aligned}$$

$$\begin{aligned} \theta_2(x, Fo) - f_{o_2} &= \frac{2}{\varepsilon Ko_m x M} \{ (N_{12} f_{o_2} - A_1 N_{22})(1 - v_1^2)(Fo)^{\frac{1}{2}} i \operatorname{erfc} \frac{(1-x)v_1}{2(Fo)^{\frac{1}{2}}} - (N_{11} f_{o_2} - A_1 N_{21} f_{o_1})(1 - v_2^2)(Fo)^{\frac{1}{2}} \\ &\quad \times i \operatorname{erfc} [v_2 \sqrt{1-x}/2(Fo)^{\frac{1}{2}}] + (N_{31} Ki_m - N_{21} Ki_q)(1 - v_2^2)(Fo)^{\frac{1}{2}} i \operatorname{erfc} [v_2 \sqrt{1-x}/2(Fo)^{\frac{1}{2}}] \\ &\quad - (N_{32} Ki_m - N_{22} Ki_q)(1 - v_1^2)(Fo)^{\frac{1}{2}} i \operatorname{erfc} [v_1 \sqrt{1-x}/2(Fo)^{\frac{1}{2}}] \} \quad (43) \end{aligned}$$

and

$$\theta_4(x, Fo) = \frac{2}{x} (Ki_{d_2} - B_3 f_{o_4})(Lu_d Fo)^{\frac{1}{2}} i \operatorname{erfc}(1 - \sqrt{2}/Lu_d Fo), \quad (44)$$

where

$$N_{1j} = (-1)^j [(B_2 + A_2 B_1)v_j + B_1(1 - v_j^2)v_j/\varepsilon Ko_m]$$

and

$$N_{2j} = [A_2 + (1 - v_j^2)/\varepsilon Ko_m]v_j, \quad N_{3j} = (-1)^{j+1}v_j$$

$$M = (v_1^2 - v_2^2)(Lu_m)^{\frac{1}{2}}/\varepsilon Ko_m.$$

The expressions for these transfer potentials at the centre can be obtained by approximating $\sinh v_j(sx) = v_j(sx)^{\frac{1}{2}}$, thus the expressions (14)–(16) become,

$$\begin{aligned} \theta_1(0, Fo) - f_{o_1} &= \frac{1}{(v_2^2 - v_1^2)} \{ [(1 - v_1^2 + \varepsilon Ko_m A_2)Ki_q - \varepsilon Ko_m Ki_m] \operatorname{erfc}[v_2/2(Fo)^{\frac{1}{2}}] \\ &\quad - [(1 - v_2^2 + \varepsilon Ko_m A_2)Ki_q - \varepsilon Ko_m Ki_m] \operatorname{erfc}[v_1/2(Fo)^{\frac{1}{2}}] \\ &\quad + [\varepsilon Ko_m B_2 f_{o_2} + (A_1 f_{o_1} - B_1 f_{o_2})(1 - v_2^2 + \varepsilon Ko_m A_2)] \operatorname{erfc}[v_1/2(Fo)^{\frac{1}{2}}] \\ &\quad - [\varepsilon Ko_m B_2 f_{o_2} + (A_1 f_{o_1} - B_1 f_{o_2})(1 - v_1^2 + \varepsilon Ko_m A_2)] \operatorname{erfc}[v_2/2(Fo)^{\frac{1}{2}}] \} \\ &\quad + Ko_d [1 - \exp(-Lu_d Ge Fo)], \quad (45) \end{aligned}$$

$$\theta_2(0, Fo) - f_{o2} = \frac{1}{\varepsilon K_{o_m}(v_2^2 - v_1^2)} \{ (1 - v_2^2)[(1 - v_1^2 + \varepsilon K_{o_m} A_2) K_{i_q} - K_{i_m}] \operatorname{erfc}[v_2/2(Fo)^{\frac{1}{2}}] - (1 - v_1^2) \times [(1 - v_2^2 + \varepsilon K_{o_m} A_2) K_{i_q} - K_{i_m}] \operatorname{erfc}[v_1/2(Fo)^{\frac{1}{2}}] + [B_2 f_{o2} + (A_1 f_{o1} - B_1 f_{o2})(1 - v_1^2 + \varepsilon K_{o_m} A_2)](1 - v_2^2) \operatorname{erfc}[v_2/2(Fo)^{\frac{1}{2}}] - [B_2 f_{o2} + (A_1 f_{o1} - B_1 f_{o2})(1 - v_2^2 + \varepsilon K_{o_m} A_2)](1 - v_1^2) \operatorname{erfc}[v_1/2(Fo)^{\frac{1}{2}}] \} \quad (46)$$

and

$$\theta_4(0, Fo) = 2Lu_d(K_{i_d2} - B_3 f_{o4}) \operatorname{erfc}[1/2(Lu_d Fo)^{\frac{1}{2}}]. \quad (47)$$

The expressions (42) and (45) contain the terms like $i \operatorname{erfc} x$ which are sufficiently small. From the expression (42), neglecting the influence of these terms, we find that the excess of temperature over the initial distribution in the process is due to chemical reaction in the body and it varies almost linearly for small values of the generalised time. From the expression (45) it can be seen that the formation of the gaseous products is directly proportional to the square root of the generalised time, Fo at the surface of the body.

Figure 1 shows the relation between $(\theta_2 - f_{o2})/K_{i_m}$ and Fo at the surface and centre of the sphere ($K_{i_m} = K_{i_q}$) under the simple boundary conditions of second kind ($A_1 = B_1 = B_2 = 0, = 0.5, K_{o_m} = 1.2, A_2 = P_n = 0.5, V_1 = 0.9437, V_2 = 1.9346$). The transfer potential of matter is unaffected with the chemical reaction for small values of the generalised time. In the small range of the generalised time, the matter is transferred from the surface speedily in comparison with the transfer of matter from the centre. Figures 2 and 3 show the distribution of matter and its gradient inside the body. From the figures, we observe that the transfer of matter from the layer nearer to the surface towards the surface occurs comparatively at a fast rate and the rate is slowed down as the layer moves farther from the surface.

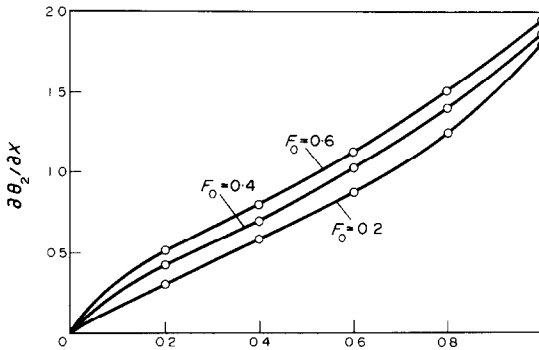


FIG. 1. Relation between $(\theta_2 - f_{o2})/K_{i_m}$ and Fo for the surface and centre of the sphere ($K_{i_m} = K_{i_q}$).

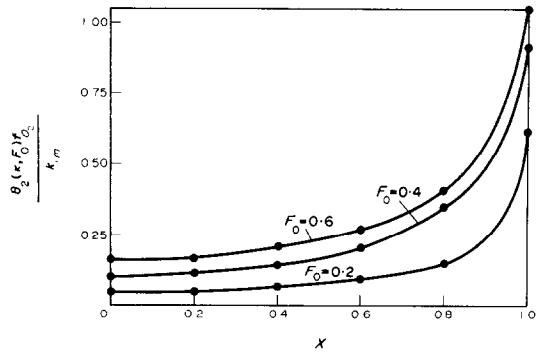


FIG. 2. Distribution of matter in a sphere ($K_{i_m} = K_{i_q}$).

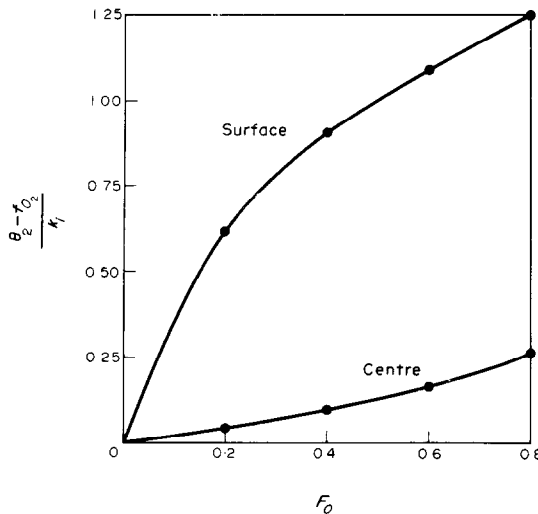


FIG. 3. Variation of gradient of matter in a sphere.

CONCLUSION

The boundary conditions appearing in the equations (9)–(11) are of most general type and describe all possible interaction laws including radiation heat effect. By particularizing these thermophysical coefficients and surface fluxes in terms of known criteria, the specific interaction law of practical interest can be obtained.

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